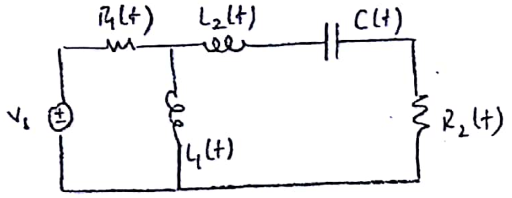
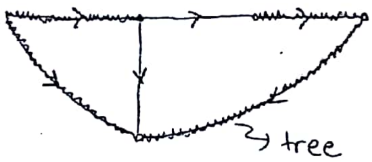
MODELING CIRCUITS with LTV / NONLINEAR ELEMENTS

Example (time-varying elements)



$\Downarrow$



Terminal equation for LTV inductor:

$$\phi(t) = L(t)\,i(t)$$

$$\Rightarrow v(t) = \frac{d}{dt}\phi(t) = L(t)\frac{di}{dt} + i(t)\frac{dL}{dt}$$

Terminal equation for LTV capacitor:

$$q(t) = C(t)\,v(t)$$

$$\Rightarrow i(t) = \frac{d}{dt}q(t) = C(t)\frac{dv}{dt} + v(t)\frac{dC}{dt}$$

$$\Rightarrow \quad \text{KCL}: \; C\dot{v}_c + \dot{C}v_c = i_{L_2}$$

$$\text{KVL}: \; L_1\dot{i}_{L_1} + \dot{L}_1 i_{L_1} - v_s + R_1(i_{L_1} + i_{L_2}) = 0$$

$$\text{KVL}: \; L_2\dot{i}_{L_2} + \dot{L}_2 i_{L_2} + v_c + R_2 i_{L_2} - v_s + R_1(i_{L_1} + i_{L_2}) = 0$$

$$\Rightarrow \dot{v}_c = -\frac{\dot{C}(t)}{C(t)}v_c + \frac{1}{C(t)}i_{L_2}$$

$$\dot{i}_{L_1} = -\frac{R_1(t) + \dot{L}_1(t)}{L_1(t)}i_{L_1} - \frac{R_1(t)}{L_1(t)}i_{L_2} + \frac{1}{L_1(t)}v_s$$

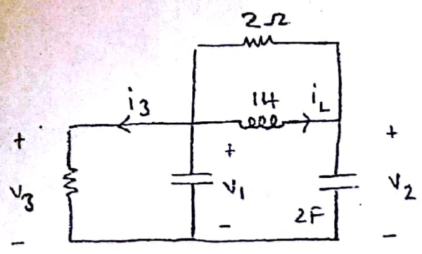
$$\dot{i}_{L_2} = -\frac{1}{L_2(t)}v_c - \frac{R_1(t)}{L_2(t)}i_{L_1} - \frac{R_1(t) + R_2(t) + \dot{L}_2(t)}{L_2(t)}i_{L_2} + \frac{1}{L_2(t)}v_s$$

Hence,

$$\begin{bmatrix} \dot{v}_c \\ \dot{i}_{L_1} \\ \dot{i}_{L_2} \end{bmatrix} = \begin{bmatrix} -\frac{\dot{C}(t)}{C(t)} & 0 & \frac{1}{C(t)} \\ 0 & -\frac{R_1(t)+\dot{L}_1(t)}{L_1(t)} & -\frac{R_1(t)}{L_1(t)} \\ -\frac{1}{L_2(t)} & -\frac{R_1(t)}{L_2(t)} & -\frac{R_1(t)+R_2(t)+\dot{L}_2(t)}{L_2(t)} \end{bmatrix} \begin{bmatrix} v_c \\ i_{L_1} \\ i_{L_2} \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{1}{L_1(t)} \\ \frac{1}{L_2(t)} \end{bmatrix} v_s$$

In general, LTV state equation reads $\boxed{\dot{x}(t) = A(t)x(t) + B(t)w(t)}$

Example (nonlinear elements)



$i_3 = \tanh(v_3)$, nonlinear resistor

$v_1 = 2(q_1 - \frac{1}{10} q_1^3)$, nonlinear capacitor

For a nonlinear capacitor (inductor) choose charge (flux) as the state variable. Let our state be $x = [q_1 \ v_2 \ i_L]^T$

$$\dot{q}_1 = -i_3 - i_L - \frac{v_1 - v_2}{2} = -\tanh(v_1) - i_L - \frac{1}{2} v_1 - \frac{1}{2} v_2$$

$$= -\tanh\left\{2q_1 - \frac{1}{5} q_1^3\right\} - i_L - q_1 + \frac{1}{10} q_1^3 - \frac{1}{2} v_2 \qquad (1)$$

$$\dot{v}_2 = \frac{1}{2}\left\{i_L + \frac{v_1 - v_2}{2}\right\} = \frac{1}{2} i_L + \frac{1}{4} v_1 - \frac{1}{4} v_2 = \frac{1}{2} i_L + \frac{1}{2}(q_1 - \frac{1}{10} q_1^3) - \frac{1}{4} v_2 \qquad (2)$$

$$\dot{i}_L = v_1 - v_2 = 2q_1 - \frac{1}{5} q_1^3 - v_2 \qquad (3)$$

(1),(2),(3) ⟹
$$\begin{bmatrix} \dot{q}_1 \\ \dot{v}_2 \\ \dot{i}_L \end{bmatrix} = \begin{bmatrix} -\tanh(2q_1 - \frac{1}{5} q_1^3) - q_1 + \frac{1}{10} q_1^3 - \frac{1}{2} v_2 - i_L \\ \frac{1}{2} q_1 - \frac{1}{20} q_1^3 - \frac{1}{4} v_2 + \frac{1}{2} i_L \\ 2q_1 - \frac{1}{5} q_1^3 - v_2 \end{bmatrix}$$

In general, nonlinear state equation reads $\boxed{\dot{x} = f(x, u)}$

(In our case, we didn't have any input and had $\dot{x} = f(x)$ with $f : \mathbb{R}^3 \to \mathbb{R}^3$)

Scanned by CamScanner